Effects of Wake Vortices on Commercial Aircraft

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Commercial aircraft are becoming more and more susceptible to wake vortex encounters at cruise altitude. Every airplane generates a trailing vortex which can linger behind the aircraft for miles. If a following aircraft penetrates the wake, it could potentially be a flight safety issue due to the induced roll on the airplane created by the circulation of the vortex. The strength of a wake vortex can be modeled and quantified by the coefficient of roll which a penetrating aircraft will experience when passing through a vortex. In this study, the coefficient was computed by a closed-form method and by using strip theory in order to analyze the effects of the vortex on the penetrating aircraft and determine the risk involved. Results showed that a penetrating aircraft with an equivalent span to the wake generating aircraft will experience a much lower coefficient of roll than when the penetrating aircraft has a smaller span. The results are especially important in a situation where there is a large disparity in aircraft size, such as a regional jet passing through the wake of a jumbo jet. Further, the coefficient of roll induced by a trailing vortex with a small core radius will be stronger than that of a vortex with a larger radius. In all cases, at least one scenario existed where the penetrating aircraft could not produce a control power coefficient of roll of the same magnitude as the induced coefficient from the vortex. Also, the taper ratio of the penetrating wing affects the strength of the induced coefficient of roll. A more rectangular wing will experience a stronger induced roll coefficient than a wing with taper. Lastly, the effect of the spanwise vortex location of the vortex on the penetrating wing was examined. The induced roll coefficient was found to be the largest when the aircraft penetrates along the vortex axis and the vortex axis coincides with the centerline of the airplane. Overall, the results determined from the various analyses offer substantial evidence to the risk of trailing wake vortex encounters at cruise altitude.

Nomenclature

AR	_	aspect ratio
	_	
a	=	vortex radius
b_{g}	=	span of the wake generating aircraft
b'_{g}	=	effective span of the wake generating aircraft
b_p	=	span of the wake penetrating aircraft
$C_{L_{lpha}}$	=	finite wing lift curve slope, $\frac{dC_L}{d\alpha}$
C_l	=	coefficient of roll
$C_{l_{control}}$	=	coefficient of roll induced by the aircraft control system
C_{l_p}	=	roll damping, $\frac{\partial C_l}{\partial \left(\frac{pb}{2U_o}\right)}$

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$C_{l_{\hat{lpha}u}}$	=	aileron control power, $\frac{dC_l}{d\delta_a}$
$C_{\ell_{\alpha}}$	=	two-dimensional wing lift curve slope
C	=	chord length
C _r	=	chord length at the wing root
C_t	=	chord length at the wing tip
dy	=	differential change in y
$F_{\rm Lift}$	=	lift force
L	=	roll moment
Q	=	dynamic pressure
S	=	wing area
V_{∞}	=	freestream velocity
$V_{ heta}$	=	tangential vortex velocity
W_{g}	=	weight of the wake generating aircraft
у	=	location on wing relative to the vortex location
α	=	angle of attack
δ_{a}	=	aileron deflection angle
Γ	=	circulation
λ	=	taper ratio, $\frac{c_t}{c_r}$
ρ	=	air density
ω	=	angular velocity

I. Introduction

TRAILING wake vortex encounters have long been studied as a potential safety hazard to commercial flight. In most instances the study is focused on interactions in the take-off and landing regions. However, now the risk for a wake vortex encounter at cruise altitude continues to grow. RVSM, or Reduced Vertical Separation Minimum, reduces the minimum vertical distance between specific cruise altitudes. Due to the evolution of commercial flight, the distance between aircraft at cruise altitudes has lessened, the amount of commercial traffic has expanded, and the disproportion in size and weight between the large fleet of commercial aircraft has grown. These changes have created a need to study and understand the possibility of wake vortex encounters at cruise altitude, and their potentially dangerous effects.¹

In general, all aircraft create a region of circulating air, or trailing vortex wake, which can linger for miles behind the generating craft. The problem with the existence of wake vortices is what can occur if an airplane passes through the wake of another airplane. Penetrating through the wake can cause several responses, although in this study, the only penetration considered is when the airplane penetrates along the vortex axis. Entering a wake vortex in this manner can prove to be the most dangerous due to the possibility of a large roll moment being induced on the aircraft from the air circulation. More specifically, this study aims to quantify the strength of the induced roll moment by calculating the induced coefficient of roll. An expression for the coefficient is derived from wing theory for both a rectangular wing and a wing with taper. Further, strip theory, a numerical integration method, is used to compute the coefficient of roll for an aircraft penetrating the wake along the vortex axis but offset from its centerline. The effects of different penetrating wing geometries are also analyzed. The overall goal of the study is to analyze the response of penetrating a wake vortex, and examining the risk involved in the response.

II. Theory

An aircraft passing through the wake vortex of a previous aircraft will experience a change in angle of attack that varies across the wing and, therefore, a roll moment will be induced. In order to quantify the strength of the induced rolling moment for comparison and analysis, the non-dimensional coefficient of roll was calculated. The coefficient was determined by two different methods. First, a closed-form analytical solution was determined. The result was based on the theory of rankine vortices and the calculation of the roll moment, and it determined the coefficient of roll for a rectangular wing and a wing with taper when the vortex was located at the center line of the wing. Second, a numerical integration method was utilized to calculate the coefficient of roll. The numerical method added depth to the analysis by allowing the coefficient to be calculated for varying vortex locations relative to the wing as well as more complicated wing geometry.

A. Vortex Model

The characteristics and theory of rankine vortices coupled with the calculation of the roll moment allows for the derivation of an expression which determines the coefficient of roll based on various aspects of the wake generating airplane and wing geometry of the penetrating airplane. The air rotating in the wake vortex has a tangential velocity which depends on the relative distance from the center of the vortex. The vortex has a core within which the tangential velocity increases linearly with increase in radius. Once outside of the vortex core, the tangential velocity begins to decrease due to an inversely proportional relationship to the radius. The tangential velocity can be expressed as follows:

$$V_{\theta} = \omega y \quad (y \le a), \tag{1}$$

and
$$V_{\theta} = \frac{\Gamma}{2\pi y}$$
 ($y \ge a$), (2)

where *a* is the core radius, and Γ is the strength of the vortex. The strength, or circulation, of the vortex is related to the weight, speed, and altitude of the generating airplane. An equation for the strength is given below, where the parameter b'_{g} is the span of the generated wake vortices.

$$\Gamma = \frac{W_g}{\rho V_\infty b'_g}.$$
(3)

To further describe the velocity profile of the vortex, the expression for ω can be determined by combining results (1) and (2) when the value of y is equal to a. After substituting the value of ω into equation (1), the new expression,

$$V_{\theta} = \frac{\Gamma}{2\pi a^2} y \quad (y \le a), \tag{4}$$

is found. A plot of the velocity profile of the vortex based on equations (2) and (4) is presented below in Figure 1.



Figure 1. The velocity profile of a rankine vortex.

B. Vortex Induced Roll Moment (Closed Form Solution)

The lift distribution across the wing of an aircraft penetrating a vortex is due to the local change in angle of attack due to the vortex velocity field.



Figure 2. Resultant velocity due to the freestream and vortex velocities.

As seen above in Figure 2, the induced angle of attack can be related as

$$\tan\left(\Delta\alpha\right) = \frac{V_{\theta}}{V_{\infty}},\tag{5}$$

and by assuming the change in angle of attack is small,

$$\Delta \alpha \approx \frac{V_{\theta}}{V_{\infty}}.$$
(6)

The spanwise change in angle of attack across the penetrating wing affects the flight of the aircraft by inducing a roll moment about the centerline of the wing. The roll moment, L, is calculated by multiplying the lift on the wing by the moment arm distance to the centerline. The moment calculation can be written in differential form,

$$dL = dF_{Lift} y , (7)$$

and from the definition of lift can be expanded to the form

$$dL = \left(C_{\ell_{\alpha}} \Delta \alpha Q c dy\right) y. \tag{8}$$

A second expression to find the roll moment involves the coefficient of roll, dynamic pressure, wing area, and the span of the aircraft. By integrating relationship (8) from zero to half of the span length and doubling the result, the value of the roll moment can be calculated. By rearranging the second moment expression, inserting the integral form of expression (8), and substituting in relationship (6), an expression for computing the coefficient of roll is derived as

$$C_{l} = \frac{1}{QSb_{p}} 2 \int_{0}^{b/2} y C_{\ell_{\alpha}} \frac{V_{\theta}(y)}{V_{\infty}} Qc(y) dy.$$
(9)

For a rectangular wing the chord length, c, is constant, and can be removed from the integral along with Q, $C_{\ell_{\alpha}}$, and V_{∞} . The coefficient can easily be calculated by separating the integral into two parts based on equations (2) and (4), which yields the result

$$C_{l} = \frac{C_{L_{\alpha}} \Gamma}{6b_{p} V_{\infty} \pi} \left(3 - 4 \frac{a}{b_{p}} \right), \tag{10}$$

where $C_{L_{\alpha}}$ is the wing lift curve slope. For a wing with taper, equation (9) can again be utilized to find the coefficient of roll, although c is no longer constant across the wing as seen in Figure 3.



Based on the geometry of the wing, the chord length of the wing as a function of the distance from the centerline is determined by

$$c(y) = c_r \left(1 + \left(\frac{\lambda - 1}{b_p / 2} \right) y \right).$$
(11)

Upon substituting the function (11) for the chord length into expression (9) for the coefficient of roll, integrating, and simplifying, the following result for calculating the coefficient is derived for a wing with taper:

$$C_{l} = \frac{C_{L_{\alpha}} \Gamma c_{r}}{12b_{p}^{2} V_{\infty} \pi} \Big(-8ab_{p} - 6(\lambda - 1)a^{2} + 6b_{p}^{2} + 3(\lambda - 1)b_{p}^{2} \Big).$$
(12)

C. Max Roll Control

After finding expressions for the coefficient of roll for both a rectangular wing and a wing with taper, it was important to determine a method which could quantify the risk involved in passing through a vortex. To do so, the maximum coefficient of roll supplied by the airplane control system was calculated for comparison to the vortex induced coefficient of roll. The control power is based on design criterion set for different categories of aircraft. For a transport, the non-dimensional roll rate should be set equal to 0.07, as seen below:

$$\left(\frac{pb}{2U_o}\right) = 0.07.$$
⁽¹³⁾

In equation (13), p is the roll rate, b is the span, and U_o is the forward speed. Further, the non-dimensional roll rate can also be presented as

$$\left(\frac{pb}{2U_o}\right) = -\frac{C_{l_{\delta a}}\delta a}{C_{l_p}},\tag{14}$$

where $C_{l_{\delta a}}$ is the aileron control power, δa is the aileron deflection angle, and C_{l_p} is the roll damping and can be calculated from

$$C_{l_p} = -\frac{C_{L_{\alpha}}}{12} \frac{\left[1+3\lambda\right]}{\left[1+\lambda\right]}.$$
(15)

The coefficient of roll created by the control system, $C_{l_{control}}$, is equivalent to the aileron control power multiplied by the aileron deflection angle. From this relationship, the coefficient of roll from the control system can be computed by rearranging equation (14) and substituting in the coefficient of roll as follows:

$$C_{l_{control}} = C_{l_{\delta a}} \delta a = -C_{l_p} \left(\frac{pb}{2U_o} \right).$$
(16)

The control system coefficient of roll will be used to normalize different plots of vortex induced coefficient results in order to find the relative strength of the vortex effects to the control power of the penetrating aircraft.

D. Strip Theory

The second method employed to calculate the coefficient of roll was a numerical integration technique known as strip theory.² The wing surface was divided into series of segments, and by using the Biot-Savart law the induced velocity normal to the surface can be found. After finding the induced velocity, the same aircraft equations of motion as above can be utilized to find the resulting roll moment coefficient induced by the vortex.

At first glance, the strip theory method seems to be inaccurate and rudimentary, although a previous study in the National Full-Scale Aerodynamics Complex (NFAC) at NASA Ames Research Center is an example which has demonstrated the validity of the method.³ A study was performed at the center which employed strip theory for estimating vortex induced loads. The experiment setup consisted of an aircraft model placed in the upstream portion of the test section along with a wing model located downstream from the aircraft model. The wing model was mounted to allow movement to different locations in the wake vortex of the generating aircraft. The induced lift and roll moment were measured in the lab, as well as the velocity distribution through the wake vortex of the generating aircraft. From the velocity distribution, the induced roll moment coefficient was computed in terms of the position in the wake. Figure 4 depicts the level of accuracy with which the strip theory method estimates the induced roll moment coefficient.



Figure 4. Predicted vortex induced rolling moment coefficient compared with experimental data.³

MATLAB was used to create a code which calculates the coefficient by separating the wing into differential areas and sums the coefficient of roll on each small area. Similar to equation (9) with differential amounts rather than an integral, C_1 is computed by

$$C_{l} = \frac{C_{L_{\alpha}} \Delta y}{Sb_{p} V_{\infty}} \sum_{i=1}^{n} V_{\theta_{i}} c_{i} y_{i} , \qquad (17)$$

The advantage of the numerical technique is the ability to find the coefficient of roll for a penetrating aircraft which flies through the wake vortex at a location other than the centerline of the aircraft. A second advantage is the ability to include more complex penetrating wing geometry. The code can place the vortex at any location along the wing and compute the resulting coefficient based on the effects of the new location on the differential areas. Further, the process can be placed inside of another iterative loop to determine the overall trend of the coefficient versus the location along the wing.

III. Results and Discussion

The theory of rankine vortices and their effect on a wing, as described above, presents detailed results when applied to specific situations. To accomplish a thorough analysis, specific values for the variables used above were gathered based on the averages for different models of aircraft as displayed in Table 1 below.

Commercial Aircraft Properties										
	Span (m)	Wing Area (m ²)	Sweep	Cr (m)	λ	Weight (kg)				
High-Capacity Aircraft	79.6	845	35.75°	17.7	0.3	350000				
Wide-Bodied Aircraft	60.5	370	31.0°	12	0.25	160000				
Twin-Jet Aircraft	36	155	25.0°	8	0.23	50000				
Regional Jet Aircraft	21.5	60	23.0°	4.5	0.26	20000				

Table 1. Collected aircraft dimensions based on different models in each category.

The data collected is based upon the published data for aircraft such as the Boeing 737, 757, 777, 787, Airbus 330, 380, Bombardier CRJ700, and Embraer ERJ-145.⁵⁻⁸ The high capacity aircraft dimensions are that of the A380 alone, and the A380 is used throughout the ensuing analysis as the wake generating aircraft. The weight displayed in the table is the landing weight of the aircraft, which is the weight of the airplane when carrying only 15 % of the maximum fuel amount.

Other variables were estimated by making assumptions and reasonable choices. For analysis where the vortex radius is held constant, it was assumed that the radius was 5 % of the span length of the wake generating aircraft. This assumption is consistent with detailed wake measurements made by NASA and the FAA. The altitude of flight in all cases was taken as 35000 feet where the air density is 0.38 kg/m³. Each airplane was assumed to be

moving at cruise velocity which was estimated at a Mach number of 0.8. The effective span of the generating aircraft used to find the circulation is determined by

$$b'_g = \frac{\pi}{4} b_g \,. \tag{18}$$

Further, the wing lift curve slope can be estimated by assuming the slope of an airfoil section and correcting for aspect ratio as shown below:

$$C_{L_{\alpha}} = \frac{5.7}{\left(1 + \frac{5.7}{\pi AR}\right)}.$$
 (19)

The first analysis was to find how C_l varies with different sized penetrating aircraft. Expression (12) was used to find the coefficient of roll for aircraft of increasing span length which were traveling through the wake of an A380. A plot was creating by calculating C_l as b_p increased from the size of a regional jet wing to that of the A380. The wing was kept geometrically similar as the span increased, so that the chord length and wing area also increased with the span. Three different trend lines were created by using the maximum take-off weight, mid-cruise weight, and the landing weight for the wake generating A380. The resulting plot is presented below in Figure 5. The span of the penetrating airplane was normalized by the span of the generating airplane to aid in viewing the results.



Figure 5. A plot of C_l for aircraft of increasing span (left). Same plot of C_l normalized by the control power (right).

As seen in Figure 5, in general C_l decreases with an increase in the span of the penetrating aircraft. Also, the C_l is larger when the generating airplane is heavier. This is due to the circulation dependence on weight, and the coefficient is directly proportional to the circulation. There is roughly a 20 % to 30 % difference in C_l based on the weight of the generating aircraft. Further, the difference due to the weight decreases as the span increases as seen by the trend lines becoming closer as the span increases in Figure 5. It becomes immediately obvious that a small spanned regional jet aircraft passing through the wake of a heavy A380 will experience a very large roll moment which can become dangerous to the airplane. For further analysis on the safety of passing through the wake vortex, the above left plot was normalized by the coefficient of roll produced by the aileron control power of the aircraft as seen above in the right plot of Figure 5. A value of one on the plot represents a vortex induced coefficient of roll produced by the control system of the plane. For each of the three A380 weights, there are penetrating span lengths which will experience a higher coefficient of roll than can be

produced by the control system of the airplane. Again, when the A380 is at a maximum weight, there is the greatest chance that the induced roll will be larger than can be controlled by the airplane. For an A380 at landing weight, the penetrating aircraft would need a span of a little over 40 % of the A380 in order to have control. However, if passing through the wake of an A380 at maximum take-off weight, the penetrating aircraft requires a span which is 75 % of the A380 span to experience a vortex induced roll which can be controlled by the aircraft. Again, safety becomes an issue. For a small regional jet with a span of 27 % of an A380 span in the wake of a maximum weight A380, the plane can experience a coefficient of roll which is well over twice of what can be controlled by the aileron control system. The control system would be rendered completely useless, and the airplane would be helplessly influenced by the vortex alone.

The second analysis focused on the change in the induced coefficient of roll due to passing through vortices with different core radii. Three trend lines were created by using three different categories of aircraft, and plotting equation (12) while varying the core radius. The resulting plot is presented below in Figure 6.



Figure 6. Plot of C_l versus vortex radius for different aircraft (left). Plot of C_l normalized by the control power versus vortex radius (right).

For each type of airplane, the core radius varied from 2.5 % of the generating span up to 25 % of the span. The core radius was then normalized by the penetrating span for ease of viewing. Again, the regional jet experiences the highest coefficient of roll. The coefficient decreases with increasing core radius in all cases. Although possibly counter-intuitive at first glance, the result stems from the inverse relationship with the core radius in expression (4). For a large radius, the maximum V_{θ} experienced by the wing would be less than that experienced in a vortex of

smaller radius. The roll moment is directly proportional to V_{θ} , and therefore, the decrease in C_l is seen above in Figure 6 as *a* increases. Similar to Figure 5, the data for increasing core radius was normalized by the control power above in the right plot of Figure 6. The normalized plot details how the effect of varying core radius compares to the control power of the airplane. A wide-body plane will not experience a coefficient of roll larger than can be produced by the control system. Both a twin-jet and regional jet can experience coefficients larger than the control system at small core radii. Although a wide-body will always have more control over the vortex, it is not a guarantee of safety. Twin-jets and regional jets are at serious risk when traveling through an A380 wake with a small core radius. While core radius is generally on the order of 5 % of the generating airplane's span, if there were a way to create a wake with a larger core radius it would be less dangerous to the trailing flights.

The final analysis based upon the closed form solution determined the effect of the taper ratio on the coefficient of roll experienced by a regional jet. Again, equation (12) was utilized in a MATLAB code to plot several discrete points with different λ , and also to fit a curve to the data. The area and aspect ratio of the regional jet wing was held constant as λ varied, so that any change would only be attributed to the taper ratio.



Figure 7. Plot of C_l for varying taper ratio of a regional jet (left). Plot of C_l as the wake vortex location traverses across the wing (right).

Figure 7 displays how C_l increases with an increase in λ . A wing with a small λ would have more area inboard, and while the maximum V_{θ} will be closer to the root, there will also be less area outboard to be affected by the larger moment arm of the vortex. A difference of roughly 25 % in C_l is between a λ of 0.1 and 1.0. Overall, a more rectangular wing will experience a higher C_l than a wing of the same area with a lower λ .

All of the previous analysis was based on a wing passing through a wake vortex which coincides with the centerline of the aircraft. In order to find the effect of a vortex not located at the centerline, the numerical integration was employed. The MATLAB code uses the basic dimensions of a small regional jet as defined above, and the wake generating airplane was an A380. By breaking the wing into differential areas and summing the coefficient of roll on each piece, the total coefficient of roll can be found for a given vortex location along the wing. A plot was created by finding the coefficient for each vortex location as it traversed from the centerline of the wing out to a location an entire span length away from the aircraft. A curve was plotted for both a wing with taper and a rectangular wing. The vortex location was normalized by half of the span, and the resulting plot is presented above in Figure 7. The C_1 is largest when the vortex is located at the center of the wing. Due to the vortex center coinciding with the center of rotation, all of the vortex velocity acts to rotate the wing in the same direction which creates the largest roll moment. As the vortex location moves outboard, the coefficient of roll quickly decreases due to a component of the vortex velocity beginning to resist the rotation originally created with the vortex at the center. When the vortex is located roughly 75 % of the half span outboard, the coefficient of roll is zero. At this point, the forces from the wake vortex on the top and bottom of the wing perfectly balance such that no roll moment is created. Once past the equilibrium location, the roll moment switches sign and begins to rotate in the opposite direction. After the roll moment switches direction, the coefficient of roll slowly returns to zero as the vortex moves further outboard. Logically, the coefficient would become zero as the vortex moves far away from the wing, as the airplane would not experience any effect from a vortex located far away from the plane. The rectangular wing experiences a roll moment of larger magnitude in general, and the zero location on the wing is slighter more outboard than for the tapered wing. The result of the numerical integration supports the theoretical result on the effect of taper as seen above in Figure 7, which stated that a more rectangular wing would experience a higher C_1 in general. For both cases, the maximum roll moment is created when passing through the vortex along the centerline, and the coefficient moves toward zero as the distance from the vortex extends to infinity.

In terms of safety, it is clear that a penetrating aircraft would be in the most danger if passing directly through a wake vortex. Although invisible to the eye, if a vortex could be avoided by just a half span length, the effect on the aircraft would diminish by nearly 70 %. There is clear danger to commercial aircraft due to the inability to view and locate a wake vortex.

IV. Conclusion

The attempt of this study was to quantify the strength of a wake vortex, and to find the overall effects on an aircraft penetrating along the vortex axis. The need for an analysis of wake vortex encounters has become increasingly necessary. The strengths of wake vortices can be quantified non-dimensionally for comparison by finding the coefficient of roll induced by the wake vortex on the penetrating aircraft. The coefficient of roll was calculated by two methods, a closed-form solution method and a numerical integration method called strip theory. The latter was utilized to move the vortex along the wing in the spanwise direction for a more in depth analysis.

Several analyses were performed in order to describe the effect of the wake vortices on the penetrating aircraft. The first was a study of the induced coefficient of roll based on the span of the penetrating aircraft. The resulting data displayed that, in general, an airplane with a larger span will experience a lower coefficient of roll than a smaller aircraft. Second, an examination of how the induced coefficient varied with vortex core radius size was completed. It was discovered that the induced coefficient of roll actually decreases for an aircraft penetrating a wake with a relatively large core radius. Also, both of the above analyses determined that there are situations which can occur frequently where the induced roll from the vortex overpowers the maximum control power of the airplane. In most cases, a regional jet type aircraft is the most susceptible to a loss of control. A third study was executed to find the effect of taper on the wake penetrating response. The coefficient of roll for a regional jet increases as the taper ratio of the aircraft wing increases. Finally, strip theory was used to find the effect of vortex location along the span on the induced coefficient faded toward zero as the distance between the airplane and vortex extended to infinity.

Overall, the evidence provided by the analytical data suggests that wake vortices are a serious hazard for commercial flight at altitude, especially along the axis of the vortex. The danger is especially prevalent for smaller regional jet aircraft, which in some scenarios might not be able to create enough control power to overcome the effects of the trailing wake.

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